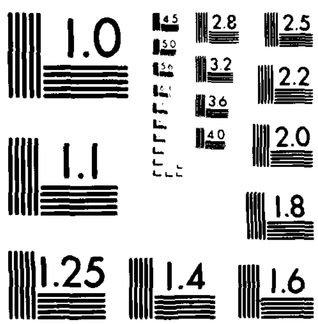
AD-A089 926 SOUTH CAROLINA UNIV COLUMBIA DEPT OF MATHEMATICS AND--ETC F/G 20/11
ON ESTIMATION OF RELIABILITY FOR THE BIRNBAUM-SAUNDERS FATIGUE --ETC(U)
JUL 80 W J PADGETT F49620-79-C-0140
UNCLASSIFIED TR-64 AFOSR-TR-80-0958 NL

1 OF 1
AD A
089926

END
DATE
FILMED
11-80
DTIC



MICROCOPY RESOLUTION TEST CHART

Department of Mathematics
and Statistics
The University of South Carolina
Columbia, South Carolina 29208

AFOSR-TR- 80-0958

F-49620-79-C-0140

ON ESTIMATION OF RELIABILITY FOR THE BIRNBAUM-SAUNDERS FATIGUE LIFE MODEL*

by

W. J. Padgett

University of South Carolina

Statistics Technical Report No. 64

62N05-5

July, 1980

Department of Mathematics and Statistics
University of South Carolina
Columbia, South Carolina 29208

*Research supported by the United States Air Force Office of Scientific Research under Contract No. F49620-79-C-0140.

ABSTRACT

The Birnbaum-Saunders fatigue life distribution with shape parameter α and scale parameter β is considered. The scale parameter β is also the median lifetime, and assuming that β is known, Bayes estimators of the reliability function are obtained for a family of proper conjugate priors as well as for Jeffreys' vague prior for α. When both α and β are unknown, a modified Bayes estimator $R_B^*(t)$ of the reliability is proposed using a moment estimator of β. In addition to being computationally simpler than the maximum likelihood estimator (mle) of reliability, Monte Carlo simulations for small samples show that $R_B^*(t)$ is better than the method of moments estimator for all α and as good as the mle for small α in the sense of mean squared errors.

Beta

R* sub B(t)

alpha

R* sub B(t)

Key Words: Fatigue life data; Bayes estimation; Method of moments; Maximum likelihood.

1. INTRODUCTION

With the relatively small amounts of fatigue life data that can be obtained for a specific type of structure, many of the two parameter lifetime models such as the gamma, lognormal, or Weibull distribution can be fitted reasonably well with respect to mean and variance. However, there is a wide discrepancy among the models for predicting the higher percentiles of the fatigue lifetime variable. Birnbaum and Saunders (1969a, 1969b) proposed a two parameter life distribution for material failure due to fatigue crack extension under cyclic loading.

The Birnbaum - Saunders fatigue life distribution has probability density function

$$f(t;\, \alpha,\beta) = (2\sqrt{2\pi}\,\alpha\,\beta\,t^2)^{-1}\,(t^2 - \beta^2)/[(t/\beta)^{\frac{1}{2}} - (\beta/t)^{\frac{1}{2}}]$$
$$\times \exp\,[-(t/\beta + \beta/t - 2)/2\alpha^2], \quad t > 0\,, \quad (1.1)$$

with $\alpha,\beta > 0$ and reliability function

$$R(t;\alpha,\beta) = \Phi\,[-((t/\beta)^{\frac{1}{2}} - (\beta/t)^{\frac{1}{2}})/\alpha], \quad t > 0\,, \qquad (1.2)$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function. The parameters α and β can be interpreted as shape and scale parameters, respectively. Also, β is the median life (Saunders, 1974) and the mean and variance are, respectively, $\mu = \beta(1 + \alpha^2/2)$ and $\sigma^2 = (\alpha\beta)^2(1 + 5\alpha^2/4)$. Other properties of the distribution are described by Birnbaum and Saunders (1969a, 1969b) or Mann, Schafer, and Singpurwalla (1974). Saunders (1974) studied a ξ-normal family of

random variables closed under reciprocation, of which the density (1.1) is a member. He discussed moment estimation as well as other estimators of β, since the maximum likelihood estimator (mle) of β cannot be found in closed form.

In this paper we consider estimation of the reliability function (1.2) by Bayesian and "modified Bayesian" techniques. Bayes estimators for a proper conjugate family of priors on α and vague priors on α will be given in Section 3, assuming β is known. This is reasonable since β is the median or "characteristic life," and information concerning β may be known. In Section 4 a modified Bayes estimator of (1.2) is proposed and compared with the mle and method of moments estimator (mme) of (1.2) for small samples. Monte Carlo simulation results indicate that the modified Bayes estimator is better than the mme but not uniformly as good as the mle. However, the mle must be computed by Newton's or other numerical iteration procedures, whereas the modified Bayes and mme are easily computed in closed form. An example is given in Section 5.

2. ESTIMATION OF α AND β

Point estimation of α and β has been investigated by Birnbaum and Saunders (1969b) and, for the more general ξ-normal family, by Saunders (1974). Their main results will be stated briefly in this section for completeness.

Let $X_1, \ldots, X_n$ denote a random sample from the pdf (1.1). Birnbaum and Saunders (1969b) showed that the maximum likelihood estimate $\hat{\beta}$ of β is the unique positive solution of $g(x) = x^2 - x[2r + K(x)] + r[s + K(x)] = 0$, where

$$r = \left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{X_i}\right)^{-1}, \qquad s = \frac{1}{n}\sum_{i=1}^{n} X_i ,$$

and

$$K(x) = \left[\frac{1}{n}\sum_{i=1}^{n}(x + X_i)^{-1}\right]^{-1}$$

are the harmonic mean, arithmetic mean, and harmonic mean function, respectively. In addition, $r < \hat{\beta} < s$. The mle of α is then $\hat{\alpha} = (s/\hat{\beta} + \hat{\beta}/r - 2)^{\frac{1}{2}}$ and the mle of reliability (1.2) is given by $R(t\ ;\ \hat{\alpha}, \hat{\beta}) = \Phi[-((t/\hat{\beta})^{\frac{1}{2}} - (\hat{\beta}/t)^{\frac{1}{2}})/\hat{\alpha}]$.

Saunders (1974) discussed several other estimators of β . First, the method of moments estimator $\beta^* = (rs)^{\frac{1}{2}}$ was shown to be a consistent estimator of β . The corresponding mme of α was $\alpha^* = (s/\beta^* + \beta^*/r - 2)^{\frac{1}{2}}$. In addition, the geometric mean $\check{\beta}_n = (\prod_{i=1}^{n} X_i)^{1/n}$ was shown to a consistent estimator of β as well as the median estimator defined by

$$\tilde{\beta}_n = \begin{cases} X_{(k+1)} & \text{if} \quad n = 2k + 1 \\ (X_{(k)}X_{(k+1)})^{\frac{1}{2}} & \text{if} \quad n = 2k , \end{cases}$$

where $X_{(k)}$ denotes the k^{th} order statistic of the sample. Thus, the corresponding estimators of (1.2), $R(t\ ;\ \alpha^*, \beta^*)$, etc., are consistent.

It is known (Birnbaum and Saunders, 1969b and Saunders, 1974) that

for small α ($\alpha < 1$) the mle $\hat{\beta}$ and the mme β^* virtually agree. However, as a result of the simulation in Section 4 of this paper (see Table 2), for $\alpha \geq 1$ the mean squared error of β^* is much larger than that of β for small samples.

3. BAYES ESTIMATION OF RELIABILITY FOR KNOWN MEDIAN LIFE

Assuming that the scale parameter and median β is known, Bayes estimators of the reliability function $R(t\,;\alpha,\beta)$ are obtained in this section with respect to squared error loss. First, a proper conjugate family of priors is used, and then a vague prior of Jeffreys is considered. A Bayesian analysis with both parameters α and β unknown seems to be mathematically intractable. Thus, a modified Bayes estimator of $R(t\,;\alpha,\beta)$ is proposed for the vague priors, where β is estimated by the moment estimators in Section 2, and is compared with the mle by Monte Carlo simulations in Section 4.

The likelihood of the random sample $x_1, \ldots, x_n$ for known β is

$$L(\alpha | x_1, \ldots, x_n) = K_1\, \alpha^{-n} \exp(-K_2/\alpha^2) \,, \tag{3.1}$$

where

$$K_1 = (2\sqrt{2\pi}\,\beta)^{-n} \prod_{i=1}^{n} \frac{x_i^2 - \beta^2}{x_i\,[(x_i/\beta)^{\frac{1}{2}} - (\beta/x_i)^{\frac{1}{2}}]}$$

and

$$K_2 = \sum_{i=1}^{n} (x_i/\beta + \beta/x_i - 2)/2 \ .$$

As a prior distribution, let α^2 have an inverted gamma distribution with parameters (ν,δ) , denoted $IG(\nu,\delta)$, and pdf

$$g(\alpha^2) = [\delta\ \Gamma\ (\nu)]^{-1}(\delta/\alpha^2)^{\nu + 1} \exp(-\delta/\alpha^2)\ , \alpha^2 > 0\ , (\nu,\delta > 0)\ ,$$

where $\Gamma(\cdot)$ denotes the usual gamma function. That is, α^{-1} is the square-root of a gamma distributed random variable. After straightforward integration of (3.1) multiplied by $g(\alpha^2)$, the posterior distribution of α^2 , given the data, is $IG(\nu + n/2\ , \delta + K_2)$. Therefore, α^{-1} , given $x_1, \ldots, x_n$, is distributed as the square-root of a gamma random variable with parameters $(\nu + n/2\ , (\delta + K_2)^{-1})$. Thus, Lemma 1 of Padgett and Wei (1977) may be applied to obtain the Bayes estimate of $R(t\ ; \alpha,\beta)$ with respect to a squared error loss function as

$$\begin{aligned} R_{PB}(t) &= E_{\alpha^{-1}|x_1, \ldots, x_n} \{\Phi[-\alpha^{-1}((t/\beta)^{\frac{1}{2}} - (\beta/t)^{\frac{1}{2}})]\} \\ &= P[T_{2\nu + n} < -\ c\ ((\nu + n/2)/(\delta + K_2))^{\frac{1}{2}}]\ , \qquad (3.2) \end{aligned}$$

where T_k is a random variable having student's t distribution with k degrees of freedom.

If there is very little information available concerning α , a vague or noninformative prior may be used to obtain a (proper) posterior distribution for α . Since the transformed random variable $Z_i = (X_i/\beta)^{\frac{1}{2}} - (\beta/X_i)^{\frac{1}{2}}$ for known β has a normal distribution with mean zero and variance α^2 for each $i=1, \ldots, n$, the Fisher's information $I_n(\alpha)$ can be found to be $-n/\alpha^2$. Thus, Jeffrey's noninformative (improper) prior $p(\alpha) \propto \alpha^{-1}$, $\alpha > 0$, is used for α (Box and Tiao,

1973). It should be noted that for the case that both α and β are unknown Box and Tiao's (1973) vague prior idea of taking the prior as $p(\beta|\alpha) \propto$ constant and $p(\alpha) \propto \alpha^{-1}$ apparently does not result in a mathematically tractable posterior distribution.

For the improper prior $p(\alpha) \propto \alpha^{-1}$, the (proper) posterior pdf of α, given $x_1, \ldots, x_n$, is

$$f(\alpha|x_1, \ldots, x_n) = \frac{2K_2^{n/2} \exp(-K_2/\alpha^2)}{\Gamma(n/2) \quad \alpha^{n+1}} \quad , \quad \alpha > 0 \quad ,$$

where K_2 is defined as before. Therefore, the Bayes estimator of $R(t\ ;\alpha,\beta)$ with respect to a squared-error loss function is given by the expectation

$$\begin{aligned} R_B(t) &= \int_0^\infty \Phi(-c/\alpha) \quad f(\alpha|x_1, \ldots, x_n)\, d\,\alpha \\ &= E_\Theta\, [\Phi(-c\,\Theta^{-\frac{1}{2}})] \quad , \end{aligned} \tag{3.3}$$

where $c = (t/\beta)^{\frac{1}{2}} - (\beta/t)^{\frac{1}{2}}$ and Θ^{-1} has a gamma distribution with parameters $(n/2\ ,\ K_2^{-1})$. Applying Lemma 1 of Padgett and Wei (1977) to the expectation in (3.3) gives the Bayes estimator as

$$R_B(t) = P[T_n < -\,c\,(n/2K_2)^{\frac{1}{2}}] \ , \ t > 0 \quad , \tag{3.4}$$

where T_n has student's t distribution with n degrees of freedom.

In the case that the median lifetime β is also unknown, but some prior information about α is known, a "modified Bayes" estimator may be obtained from (3.2) by choosing appropriate inverted gamma prior

parameters and using one of the estimators of β given in Section 2. Also, in the case of unknown β with vague prior information about α a modified Bayes estimator may be obtained from (3.4) by replacing β with $\hat{\beta}$, β^*, or another estimator from Section 2. In the next section, the behavior of this second modified Bayes estimator is indicated by Monte Carlo simulation results when β^* is used for β in (3.4). Denote this estimator by $R_B^*(t)$. The simulations indicate that $R^*(t)$ is generally better than the mme $R^*(t) = R(t\ ;\ \alpha^*, \beta^*)$ for all α and as good as the mle $\hat{R}(t) = R(t\ ;\ \hat{\alpha}, \hat{\beta})$ only when $\alpha \leq 1$. However, the drawback of the mle is that $\hat{\beta}$ must be approximated iteratively as discussed in Section 2.

4. SMALL SAMPLE COMPARISONS OF ESTIMATORS OF RELIABILITY

For small samples direct analytical comparisons of the mle $R(\hat{t})$, $R^*(t)$, and modified Bayes estimator $R_B^*(t)$ given in Sections 2 and 3 are not feasible due to the mathematical complexity of these estimators. Hence, Monte Carlo simulations were performed to compare the biases and mean squared errors of the estimators. For several values of the parameters α and β, 1000 random samples of size $n(= 10, 30)$ were generated from (1.1). For each sample the mle $\hat{\alpha}$, $\hat{\beta}$, and $\hat{R}(t)$, the mme α^*, β^*, and $R^*(t)$, and the modified Bayes estimate $R_B^*(t)$ were computed for several values of time t. The average squared error (ASE) and average estimated reliability (AER) for each of the three estimates of $R(t)$ were computed over the 1000 samples. Table 1 shows some of the simulation results. Since β is a scale parameter the results in Table 1 do not depend on the specific value of β. Time t was chosen as multiples of β. The same results were obtained when 2000 samples were used to generate the ASE and AER instead of 1000.

TABLE 1. Averages and ASE($\times 10^{-2}$) of Estimators of $R(t;\alpha,\beta)$
(times $t_i = i\beta/2$, $i = 1, 2, 3, 4$)

(a) $n = 10$

α	Time	True $R(t;\alpha,\beta)$	MLE $\hat{R}(t)$ AER	MLE $\hat{R}(t)$ ASE	MME $R^*(t)$ AER	MME $R^*(t)$ ASE	Mod. Bayes $R_B^*(t)$ AER	Mod. Bayes $R_B^*(t)$ ASE
0.10	t_1	1.000	1.000	0.000	1.000	0.000	1.000	0.000
	t_2	0.500	0.488	2.011	0.488	2.011	0.489	1.893
0.25	t_1	0.998	0.995	0.008	0.995	0.008	0.990	0.022
	t_2	0.500	0.488	2.001	0.488	2.000	0.489	1.883
	t_3	0.051	0.051	0.209	0.051	0.209	0.063	0.234
0.50	t_1	0.921	0.924	0.382	0.924	0.385	0.911	0.391
	t_2	0.500	0.488	1.964	0.488	1.964	0.488	1.845
	t_3	0.207	0.198	1.032	0.198	1.032	0.208	0.949
1.00	t_1	0.760	0.776	1.140	0.776	1.172	0.767	1.071
	t_2	0.500	0.488	1.820	0.489	1.841	0.489	1.733
	t_3	0.342	0.335	1.437	0.334	1.456	0.339	1.355
	t_4	0.240	0.221	1.190	0.220	1.212	0.229	1.104
5.00	t_1	0.556	0.566	0.542	0.569	0.776	0.567	0.727
	t_2	0.500	0.493	0.649	0.493	0.869	0.493	0.823
	t_3	0.467	0.466	0.505	0.465	0.711	0.466	0.670
	t_4	0.444	0.431	0.588	0.423	0.849	0.430	0.795

TABLE 1. (cont'd)

(b) $n = 30$

α	Time	True $R(t;\alpha,\beta)$	MLE $\hat{R}(t)$ AER	MLE $\hat{R}(t)$ ASE	MME $R^*(t)$ AER	MME $R^*(t)$ ASE	Mod. Bayes $R_B^*(t)$ AER	Mod. Bayes $R_B^*(t)$ ASE
0.25	t_1	0.998	0.997	0.002	0.997	0.002	0.995	0.003
	t_2	0.500	0.501	0.533	0.501	0.532	0.501	0.523
	t_3	0.051	0.052	0.081	0.052	0.081	0.057	0.086
0.50	t_1	0.921	0.923	0.136	0.923	0.136	0.918	0.136
	t_2	0.500	0.501	0.514	0.501	0.514	0.501	0.505
	t_3	0.207	0.204	0.348	0.205	0.350	0.208	0.339
1.00	t_1	0.760	0.765	0.366	0.765	0.377	0.762	0.364
	t_2	0.500	0.501	0.444	0.501	0.455	0.501	0.447
	t_3	0.342	0.339	0.418	0.339	0.440	0.340	0.429
	t_4	0.240	0.236	0.376	0.237	0.392	0.240	0.380
3.00	t_1	0.593	0.597	0.185	0.597	0.258	0.596	0.253
	t_2	0.500	0.501	0.160	0.501	0.226	0.501	0.222
	t_3	0.446	0.444	0.163	0.444	0.245	0.444	0.241
	t_4	0.407	0.403	0.186	0.404	0.271	0.405	0.265
5.00	t_1	0.556	0.559	0.078	0.559	0.140	0.558	0.138
	t_2	0.500	0.501	0.065	0.501	0.119	0.501	0.117
	t_3	0.467	0.466	0.068	0.466	0.132	0.466	0.130

TABLE 2. ASE of Estimators of α and β for $n = 30$.

(α, β)	MLE $\hat{\alpha}$	MLE $\hat{\beta}$	MME α^*	MME β^*
(0.1, 30)	0.0163[1]	0.290	0.0163[1]	0.290
(0.25, 50)	0.1018	4.968	0.1018	4.972
(0.50, 50)	0.4381	19.517	0.4382	19.536
(1.0, 200)	1.6420	1033.9	1.6444	1086.3
(3.0, 200)	15.262	3106.3	15.257	5143.3
(5.0, 100)	45.897	1013.7	45.973	2315.4

[1] ASE $\times 10^{-2}$ for α

Table 2 shows some of the ASE's for the mle and mme of α and β. As indicated in Section 2, the estimator β^* is in agreement with $\hat{\beta}$ for $\alpha < 1$, but differs from $\hat{\beta}$ considerably in mean squared error for $\alpha \geq 1$.

As the simulation results of Table 1 indicate, the modified Bayes estimator $R_B^*(t)$ is better in mean squared error than the moment estimator $R^*(t)$ for all values of α. It also is no worse than the mle for $\alpha \leq 1$, although the performance of the mle is better than that of $R_B^*(t)$ for $\alpha > 1$. The biases of all three estimators are similar for all values of α.

In addition to using the moment estimator β^* to modify the Bayes estimator (3.4), the other consistent closed form estimators of β, $\check{\beta}_n = (\prod_{i=1}^{n} X_i)^{1/n}$ and the median $\tilde{\beta}_n$, stated in Section 2 were used. These produced slightly larger ASE than $R_B^*(t)$ did. Also, if it is known that α is near a specific value, one of the proper inverted gamma priors of Section 3 could be used to produce a modified Bayes estimator from (3.2). This would generally be a better estimator than $R_B^*(t)$.

5. EXAMPLE

As an example, some of the fatigue failure data of Birnbaum and Saunders (1958) will be used. Specifically, the failure times for 101 strips of 6061-T6 aluminum sheeting which were recorded under testing with periodic loading at 18 cycles per second and maximum stress of 31,000 psi will be used to estimate the reliability by $\hat{R}(t)$, $R^*(t)$, and $R_B^*(t)$. This data is known to follow approximately the distribution (1.1).

Newton's method yielded the mle of β as $\hat{\beta} = 1336.37$, and the mle of α was $\hat{\alpha} = 0.31032$. The moment estimator of β was $\beta^* = 1340.86$, and $\alpha^* = 0.31034$. Table 3 gives the estimates of reliability $R(t\ ;\ \alpha,\ \beta)$ for several values of t .

TABLE 3. RELIABILITY ESTIMATION FOR ALUMINUM SHEETING

t	600	800	1000	1200	1400	1600	1800	2000
$\hat{R}(t)$	0.9960	0.9527	0.8258	0.6357	0.4404	0.2806	0.1677	0.0954
$R^*(t)$	0.9961	0.9538	0.8286	0.6398	0.4447	0.2843	0.1705	0.0973
$R^*_B(t)$	0.9955	0.9522	0.8274	0.6394	0.4448	0.2849	0.1716	0.0988

6. SUMMARY AND CONCLUSIONS

For the case that the median life (or scale parameter) β is known, Bayes estimators of the reliability function for the Birnbaum-Saunders model have been presented with respect to squared error loss functions and a proper conjugate family of priors (square root of gamma) or a vague prior. If both parameters α and β are unknown, Bayes solutions for reliability in a compact form seem to be extremely difficult. Hence, an appealing modified Bayes estimator $R^*_B(t)$ is proposed in which the value of β is replaced by the moment estimate β^* in the Bayes solution (3.4). This modified Bayes estimator is easily computed from Student's t distribution whereas the maximum likelihood estimator of reliability must be computed by an iteration procedure for the mle $\hat{\beta}$ of β . The modified Bayes estimator has been shown by computer simulation results in Section 4 to be preferable in the sense of mean squared error to the

method of moments estimator $R^*(t)$ for all values of α . In addition to being computationally simpler than the mle, the simulations indicate that $R_B^*(t)$ is as good as the mle in mean squared error for $\alpha \leq 1$. The biases of all three of the estimators of reliability are approximately the same.

It should also be remarked that minimum variance unbiased estimators of α , β , or $R(t ; \alpha, \beta)$ are not available due to the difficulty in obtaining jointly sufficient complete statistics for α and β . The pdf (1.1) is not of the exponential form.

ACKNOWLEDGEMENT

The author is grateful to Professor John Spurrier for helpful discussions concerning this paper.

REFERENCES

Birnbaum, Z. W. and Saunders, S. C. (1958). A statistical model for life length of material. J. Amer. Statist. Assoc., 53, 151-160.

Birnbaum, Z. W. and Saunders, S. C. (1969a). A new family of life distributions. Journal of Applied Probability, 6, 319-327.

Birnbaum, Z. W. and Saunders, S. C. (1969b). Estimation for a family of life distributions with applications to fatigue. Journal of Applied Probability, 6, 328-347.

Box, G. E. P. and Tiao, G. C. (1973). Bayesian Inference in Statistical Analysis. Reading, MA: Addison-Wesley.

Mann, N. R., Schafer, R. E. and Singpurwalla, N. D. (1974). Methods for Statistical Analysis of Reliability and Life Data. New York: John Wiley & Sons.

Padgett, W. J. and Wei, L. J. (1977). Bayes estimation of reliability for the two-parameter lognormal distribution. Communications in Statistics - Theory and Methods, A6, 443-457.

Saunders, S. C. (1974). A family of random variables closed under reciprocation. J. Amer. Statist. Assoc., 69, 533-539.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR-80-0958	2. GOVT ACCESSION NO. AD-A089 926	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ON ESTIMATION OF RELIABILITY FOR THE BIRNBAUM-SAUNDERS FATIGUE LIFE MODEL		5. TYPE OF REPORT & PERIOD COVERED Interim
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) W. J. Padgett		8. CONTRACT OR GRANT NUMBER(s) F49620-79-C-0140
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of South Carolina, Department of Mathematics and Statistics, Columbia, South Carolina 29208		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304/A5
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM, Bolling AFB, Washington, D.C. 20332		12. REPORT DATE July 1980
		13. NUMBER OF PAGES 17
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fatigue life data; Bayes estimation; Method of moments; Maximum likelihood.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The Birnbaum-Saunders fatigue life distribution with shape parameter α and scale parameter β is considered. The scale parameter β is also the median lifetime, and assuming that β is known, Bayes estimators of the reliability function are obtained for a family of proper conjugate priors as well as for Jeffrey's vague prior for α. When both α and β are unknown, a modified Bayes estimator $R_B^*(t)$ of the reliability is proposed using a moment estimator of β. In addition to being computationally simpler than the maximum likelihood estimator (mle) of reliability, Monte Carlo simulations for small samples

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

41044 2

JOB

20. (continued)

show that $R_B^*(t)$ is better than the method of moments estimator for all α and as good as the mle for small α in the sense of mean squared errors.